

# Vacuum stability in neutrinophilic Higgs doublet model

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## Abstract

A neutrinophilic Higgs model has tiny vacuum expectation value (VEV), which can naturally explain tiny masses of neutrinos. There is a large energy scale hierarchy between a VEV of the neutrinophilic Higgs doublet and that of usual standard model-like Higgs doublet. In this paper we at first analyze vacuum structures of Higgs potential in both supersymmetry (SUSY) and non-SUSY neutrinophilic Higgs models, and next investigate a stability of this VEV hierarchy against radiative corrections. We will show that the VEV hierarchy is stable against radiative corrections in both Dirac neutrino and Majorana neutrino scenarios in both SUSY and non-SUSY neutrinophilic Higgs doublet models.

# 1 Introduction

The recent neutrino oscillation experiments gradually reveal a structure of lepton sector[1, 2]. However, from the theoretical point of view, smallness of neutrino mass is still a mystery and it is one of the most important clues to find new physics beyond the standard model (SM). A lot of ideas have been suggested to explain the smallness of neutrino masses comparing to those of quarks and charged leptons. How about considering a possibility that the smallness of the neutrino masses is originating from an extra Higgs doublet with a tiny vacuum expectation value (VEV). This idea is that neutrino masses are much smaller than other fermions because the origin of them comes from different VEV of different Higgs doublet, and then we do not need extremely tiny neutrino Yukawa coupling constants. This kind of model is so-called neutrinophilic Higgs doublet model [3]-[13], where a neutrinophilic Higgs take a VEV of  $\mathcal{O}(0.1)$  eV in Dirac neutrino scenario[5, 6, 8, 9], while a VEV of  $\mathcal{O}(1)$  MeV in Majorana neutrino scenario with TeV-scale seesaw[3, 4, 7, 10, 11, 12, 13]. The non-supersymmetric (non-SUSY) neutrinophilic Higgs doublet model is sometimes called  $\nu$ THDM. The (collider) phenomenology in  $\nu$ THDM is interesting, since a charged Higgs boson is almost originated from the extra neutrinophilic Higgs doublet and its couplings to neutrinos are not small. The characteristic signals of the  $\nu$ THDM could be detected at LHC and ILC experiments[9, 11]. Not small neutrino Yukawa couplings in the  $\nu$ THDM can also make low energy thermal leptogenesis work[12]. The SUSY version of neutrinophilic Higgs doublet model have been suggested in Refs.[12, 13], where a thermal leptogenesis in a low energy scale works without gravitino problem[12, 13]\*.

Anyhow, a neutrinophilic Higgs model has tiny VEV, and there is a large energy scale hierarchy between a VEV of the neutrinophilic Higgs doublet and that of usual SM-like Higgs doublet. In this paper, we at first analyze vacuum structures of Higgs potential in both SUSY and non-SUSY neutrinophilic Higgs models, and next investigate a stability of this VEV hierarchy against radiative corrections. We will show that the VEV hierarchy is stable against radiative corrections in both Dirac neutrino and Majorana neutrino scenarios in both SUSY and non-SUSY neutrinophilic Higgs doublet models.

## 2 $\nu$ THDM

Let us analyze vacuum structures of Higgs potential in non-SUSY neutrinophilic Higgs model, i.e.,  $\nu$ THDM at first, and next investigate a stability of this VEV hierarchy against radiative corrections.

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\*Cosmological constraints were argued in Ref.[14], however, a setup of them is different from usual neutrinophilic Higgs doublet models, since it includes a light Higgs particle.

## 2.1 Vacuum structure in tree-level potential

We here overview the  $\nu$ THDM, where we introduce a neutrinophilic Higgs doublet  $\Phi_\nu$  and  $Z_2$ -parity as follows.

| Fields                          | $Z_2$ -parity | Lepton number                |
|---------------------------------|---------------|------------------------------|
| SM Higgs $\Phi$                 | +             | 0                            |
| neutrinophilic Higgs $\Phi_\nu$ | −             | 0                            |
| right-handed neutrino           | −             | 1                            |
| others                          | +             | $\pm 1$ : leptons, 0: quarks |

Yukawa interactions are given by

$$\mathcal{L}_{\text{Yukawa}} = y^u \bar{Q}_L \Phi U_R + y^d \bar{Q}_L \tilde{\Phi} D_R + y^l \bar{L} \Phi E_R + y^\nu \bar{L} \Phi_\nu N + \text{h.c.}, \quad (2.1)$$

where  $\tilde{\Phi} = i\sigma_2 \Phi$ , and generation indexes are omitted. Note that the right-handed neutrino only couples with  $\Phi_\nu$  through the Yukawa interaction, and this is the origin of smallness of the neutrino masses. When we include Majorana mass of right-handed neutrinos  $\frac{1}{2} M \bar{N}^c N$ , this model becomes Majorana neutrino scenario through the seesaw mechanism[15]. A Higgs potential is given by

$$\begin{aligned} V^{\nu\text{THDM}} = & -m_1^2 \Phi^\dagger \Phi + m_2^2 \Phi_\nu^\dagger \Phi_\nu - m_3^2 (\Phi^\dagger \Phi_\nu + \Phi_\nu^\dagger \Phi) + \frac{\lambda_1}{2} (\Phi^\dagger \Phi)^2 + \frac{\lambda_2}{2} (\Phi_\nu^\dagger \Phi_\nu)^2 \\ & + \lambda_3 (\Phi^\dagger \Phi) (\Phi_\nu^\dagger \Phi_\nu) + \lambda_4 (\Phi^\dagger \Phi_\nu) (\Phi_\nu^\dagger \Phi) + \frac{\lambda_5}{2} [(\Phi^\dagger \Phi_\nu)^2 + (\Phi_\nu^\dagger \Phi)^2], \end{aligned} \quad (2.2)$$

where parameters are  $m_1 \sim m_2 \sim \mathcal{O}(100)$  GeV and  $\lambda_i \sim \mathcal{O}(1)$  ( $i = 1, \dots, 5$ ). As for a magnitude of  $|m_3^2|$ , we take  $(\mathcal{O}(10^{-0.5}) \text{ GeV})^2$  for Majorana neutrino scenario, and  $(\mathcal{O}(10^{-1}) \text{ MeV})^2$  for Dirac neutrino scenario. Notice that  $\Phi$  has negative mass squared as  $(-m_1^2) < 0$ . The Higgs doublets are assumed to be take real VEVs as  $\langle \Phi \rangle = (v_1, 0)^T$  and  $\langle \Phi_\nu \rangle = (v_2, 0)^T$ , then, stationary conditions are given by

$$0 = \frac{1}{2} \frac{\partial V^{\nu\text{THDM}}}{\partial v_1} = -m_1^2 v_1 - m_3^2 v_2 + \lambda_1 v_1^3 + \hat{\lambda} v_1 v_2^2, \quad (2.3)$$

$$0 = \frac{1}{2} \frac{\partial V^{\nu\text{THDM}}}{\partial v_2} = m_2^2 v_2 - m_3^2 v_1 + \lambda_2 v_2^3 + \hat{\lambda} v_1^2 v_2, \quad (2.4)$$

where  $\hat{\lambda} \equiv \lambda_3 + \lambda_4 + \lambda_5$ . We sort the following three cases by magnitude relations between  $|v_1|$  and  $|v_2|$ .

1.  $|v_2| \ll |v_1|$  case: This vacuum is what the  $\nu$ THDM wants to realize. The magnitudes of VEVs are given by

$$|v_1| \simeq \sqrt{\frac{m_1^2}{\lambda_1}}, \quad v_2 \simeq \frac{m_3^2 v_1}{m_2^2 + \hat{\lambda} v_1^2}, \quad (2.5)$$

and a potential height at the vacuum is given by

$$V_{|v_2| \ll |v_1|}^{\text{THDM}} \simeq -m_1^2 v_1^2 + \frac{\lambda_1}{2} v_1^4 \simeq -\frac{m_1^4}{2\lambda_1}.$$

2.  $|v_1| \ll |v_2|$  case: This vacuum suggests  $v_2(m_2^2 + \lambda_2 v_2^2) = 0$  from Eq.(2.3), and thus,

$$v_2^2 = \begin{cases} 0 & (m_2^2 > 0), \\ -\frac{m_2^2}{\lambda_2} & (m_2^2 < 0). \end{cases} \quad (2.6)$$

The case of  $v_2^2 = 0$  contradicts  $|v_1| \ll |v_2|$ . Another case of  $v_2^2 = -m_2^2/\lambda_2$  suggests the value of  $v_1$  as  $v_1^2 = m_3^2 v_2^2 / (-m_1^2 + \hat{\lambda} v_2^2)^2 > 0$ , where a potential height is given by

$$V_{|v_2| \gg |v_1|}^{\nu\text{THDM}} \simeq -\frac{m_2^4}{2\lambda_2}. \quad (2.7)$$

3.  $|v_1| \sim |v_2|$  case: Neglecting tiny parameter  $m_3^2$ , the stationary conditions Eqs.(2.3) and (2.4) become

$$-m_1^2 v_1 + \lambda_1 v_1^3 + \hat{\lambda} v_1 v_2^2 = 0, \quad (2.8)$$

$$m_2^2 v_2 + \lambda_2 v_2^3 + \hat{\lambda} v_1^2 v_2 = 0. \quad (2.9)$$

Then, VEVs are given by

$$v_1^2 \simeq -\frac{\lambda_2 m_1^2 + \hat{\lambda} m_2^2}{\hat{\lambda}^2 - \lambda_1 \lambda_2}, \quad v_2^2 \simeq \frac{\hat{\lambda} m_1^2 + \lambda_1 m_2^2}{\hat{\lambda}^2 - \lambda_1 \lambda_2}, \quad (2.10)$$

and the potential height at the vacuum is estimated as

$$V_{v_1 \sim v_2}^{\nu\text{THDM}} \simeq \frac{\lambda_1 m_2^4 + \lambda_2 m_1^4 + 2\hat{\lambda} m_1^2 m_2^2}{2(\hat{\lambda}^2 - \lambda_1 \lambda_2)}. \quad (2.11)$$

Notice that the  $\nu\text{THDM}$  wants to realize the vacuum in Eq.(2.5) so that this vacuum at  $|v_2| \ll |v_1|$  should be a global minimum. Conditions of  $V_{|v_2| \ll |v_1|}^{\nu\text{THDM}} < V_{|v_1| \sim |v_2|}^{\nu\text{THDM}}$  or  $V_{|v_2| \gg |v_1|}^{\nu\text{THDM}} < V_{|v_1| \sim |v_2|}^{\nu\text{THDM}}$  suggest

$$\hat{\lambda}^2 = (\lambda_3 + \lambda_4 + \lambda_5)^2 > \lambda_1 \lambda_2. \quad (2.12)$$

This is a necessary condition for  $v_1 \gg v_2$  to be the global minimum, and an additional condition  $-\frac{m_1^4}{2\lambda_1} < -\frac{m_2^4}{2\lambda_2}$  makes the vacuum true global minimum. For the potential to be bounded from below[10, 16], quartic terms must satisfy

$$\sqrt{\lambda_1 \lambda_2} > -(\lambda_3 + \lambda_4 \pm \lambda_5), \quad \sqrt{\lambda_1 \lambda_2} > -\lambda_3, \quad \lambda_1 > 0, \quad \lambda_2 > 0. \quad (2.13)$$

These are the conditions of bounded below of the Higgs potential. We can show that a case of  $\hat{\lambda} < 0$  cannot satisfy the global minimum condition. Therefore, only a case of  $\hat{\lambda} > 0$  can satisfy the global minimum condition. Thus, in order for the desirable vacuum  $v_1 \gg v_2$  to be the global minimum, a condition

$$0 < \sqrt{\lambda_1 \lambda_2} < \hat{\lambda}, \quad \sqrt{\lambda_1 \lambda_2} > -(\lambda_3 + \lambda_4 - \lambda_5), \quad \lambda_1, \lambda_2 > 0 \quad (2.14)$$

is needed.

Next, let us estimate a curvature (mass squared) at each vacuum, which is given by

$$M_{ij}^2 = \frac{1}{2} \frac{\partial^2 V^{\text{THDM}}}{\partial v_i \partial v_j} = \begin{pmatrix} -m_1^2 + 3\lambda_1 v_1^2 + \hat{\lambda} v_2^2 & -m_3^2 + 2\hat{\lambda} v_1 v_2 \\ -m_3^2 + 2\hat{\lambda} v_1 v_2 & m_2^2 + 3\lambda_2 v_2^2 + \hat{\lambda} v_1^2 \end{pmatrix}. \quad (2.15)$$

Then, the eigenvalue equation (eigenvalue:  $x$ ) is given by

$$\begin{aligned} x^2 - (-m_1^2 + m_2^2 + (3\lambda_1 + \hat{\lambda})v_1^2 + (3\lambda_2 + \hat{\lambda})v_2^2)x - m_1^2 m_2^2 - m_3^4 + 3\hat{\lambda}(\lambda_1 v_1^4 + \lambda_2 v_2^4) \\ + (3\lambda_1 m_2^2 - \hat{\lambda} m_1^2)v_1^2 - (3\lambda_2 m_1^2 - \hat{\lambda} m_2^2)v_2^2 + 3(3\lambda_1 \lambda_2 - \hat{\lambda}^2)v_1^2 v_2^2 + 4m_3^2 \hat{\lambda} \lambda_1 \lambda_2 = 0, \end{aligned} \quad (2.16)$$

and we can estimate the curvature for above three cases.

1.  $|v_1| \gg |v_2|$  case: The eigenvalue equation in Eq.(2.16) becomes

$$x^2 - (-m_1^2 + m_2^2 + (3\lambda_1 + \hat{\lambda})v_1^2)x + 3\hat{\lambda}\lambda_1 v_1^4 + (3\lambda_1 m_2^2 - \hat{\lambda} m_1^2)v_1^2 - m_1^2 m_2^2 \simeq 0. \quad (2.17)$$

By using Eq.(2.5), it becomes

$$(x - 2m_1^2)(x - \hat{\lambda}v_1^2 + m_2^2) = 0, \quad (2.18)$$

which means

$$x = 2m_1^2, \quad \hat{\lambda}v_1^2 + m_2^2. \quad (2.19)$$

Thus,  $m_1^2 > 0$  and  $\hat{\lambda}m_1^2 + \lambda_1 m_2^2$  must be needed for  $x > 0$ .

2.  $|v_1| \ll |v_2|$  case: Using  $v_2^2 = -\frac{m_2^2}{\lambda_2}$  in Eq.(2.6), the eigenvalue equation in Eq.(2.16) becomes

$$(x + 2m_2^2)(x - (\hat{\lambda}v_2^2 - m_1^2)) = 0. \quad (2.20)$$

Thus, the solution is given by

$$x = -2m_2^2, \quad \hat{\lambda}v_2^2 - m_1^2, \quad (2.21)$$

which means  $m_2^2 < 0$ ,  $\hat{\lambda}m_2^2 + \lambda_2 m_1^2 < 0$  for  $x > 0$ .

3.  $|v_1| \sim |v_2|$  case: By neglecting  $m_3^2$  and using Eq.(2.10), the eigenvalue equation in Eq.(2.16) becomes

$$x^2 - 2(\lambda_1 v_1^2 + \lambda_2 v_2^2)x - 4(\hat{\lambda}^2 - \lambda_1 \lambda_2)v_1^2 v_2^2 = 0, \quad (2.22)$$

which means two eigenvalues  $x_1, x_2$  should satisfy

$$x_1 + x_2 = 2(\lambda_1 v_1^2 + \lambda_2 v_2^2), \quad x_1 x_2 = -4(\hat{\lambda}^2 - \lambda_1 \lambda_2)v_1^2 v_2^2. \quad (2.23)$$

Since positive  $x_1, x_2$  give positive  $x_1 + x_2, x_1 x_2$ , the positive curvature condition at this vacuum is given by

$$\lambda_1 v_1^2 + \lambda_2 v_2^2 > 0, \quad (2.24)$$

$$-(\hat{\lambda}^2 - \lambda_1 \lambda_2)v_1^2 v_2^2 > 0. \quad (2.25)$$

Thus,  $\hat{\lambda}^2 - \lambda_1 \lambda_2 < 0$ , is a positive curvature condition at the vacuum of  $|v_1| \sim |v_2|$ .

The squared masses of the charged Higgs and of the pseudoscalar must be also positive. These conditions are equivalent to

$$m_2^2 + \lambda_2 v_2^2 + \lambda_3 v_1^2 > 0 \quad (2.26)$$

$$m_2^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v_1^2 > 0. \quad (2.27)$$

Summarizing conditions for the vacuum we want, at first,  $\hat{\lambda}^2 - \lambda_2 \lambda_2$  must be positive for the vacua of  $|v_1| \gg |v_2|$  and  $|v_1| \ll |v_2|$  to be lower than that of  $|v_1| \sim |v_2|$ , and  $-\frac{m_1^4}{2\lambda_1} < -\frac{m_2^4}{2\lambda_2}$  makes the vacuum of  $|v_1| \gg |v_2|$  the global minimum. Note that  $\hat{\lambda}$  must be also positive to be consistent with the conditions of the potential bounded from below. Next, positive curvature conditions are  $m_2^2 > 0$  or  $\hat{\lambda}m_1^2 + \lambda_1 m_2^2 > 0$  with  $m_2^2 < 0$ . Finally, positive curvature of the charged Higgs and the pseudoscalar components require  $m_2^2 + \lambda_3 v_1^2 > 0$  and  $m_2^2 + (\lambda_3 + \lambda_4 - \lambda_5)v_1^2 > 0$  at  $|v_1| \gg |v_2|$ . In Table 1, we show which vacuum becomes the global minimum depending on signs of  $m_2^2$ ,  $\hat{\lambda}$ , and  $\hat{\lambda}^2 - \lambda_1 \lambda_2$ .

Can a “local minimum” at  $|v_2| \ll |v_1|$  in (2), (4) and (5) be our vacuum? It might be possible if a life time of the local minimum is long enough. There is a transition process from the local minimum at  $|v_2| \ll |v_1|$  to the global minimum at  $|v_1| \sim |v_2|$ . Its transition probability of tunneling rate suggests the life time is much shorter than an age of our universe, since a “distance” and a “height” of wall between the local and global minimums are both  $\mathcal{O}(100)$  GeV with  $\mathcal{O}(1)$  couplings of  $\lambda_i$  in Higgs potential. So, unfortunately, the local minimum cannot be our vacuum. Therefore, in the  $\nu$ THDM, we must use the suitable parameter setup as (1) or (3) with  $-\frac{m_1^4}{2\lambda_1} < -\frac{m_2^4}{2\lambda_2}$ .

Before closing this subsection, we comment on recent analyzes of vacuum structure in general THDM. For example, in Ref.[16], they investigated the vacuum instability of charge and/or CP breakings at tree level. As for so-called Inert Doublet Model (IDM) [17], it has exact  $Z_2$ -symmetry with  $m_3^2 = 0$ . This Inert Doublet does not couple with any matter fermions, which is crucial difference from our model.

|     | $(m_2^2, \hat{\lambda})$ | $\hat{\lambda}^2 - \lambda_1 \lambda_2$ | $ v_1  \gg  v_2 $ |     | $ v_1  \sim  v_2 $ |    | $ v_1  \ll  v_2 $ |     |
|-----|--------------------------|---|-------------------|-----|--------------------|----|-------------------|-----|
|     |                          |   | GM                | PC  | GM                 | PC | GM                | PC  |
| (1) | (+,+)                    | +                                       | ✓                 | ✓   |                    |    | ✓                 |     |
| (2) | (+,+)                    | -                                       |                   | ✓   | ✓                  | ✓  |                   |     |
| (3) | (-,+)                    | +                                       | ✓                 | (a) |                    |    | ✓                 | (b) |
| (4) | (-,+)                    | -                                       |                   | (a) | ✓                  | ✓  |                   | (b) |
| (5) | (+,-)                    | -                                       |                   | (a) | ✓                  | ✓  |                   |     |
| (6) | (-,-)                    | -                                       |                   |     | ✓                  | ✓  |                   |     |

Table 1: Six cases which satisfy conditions in Eq.(2.13), (2.26) and (2.27). Here GM means “Global Minimum” and PC means “Positive Curvature”, and ✓ in GM (PC) says each vacuum can be the global minimum (has positive curvature). (a) and (b) mean that the positive curvature requires conditions of (a):  $\hat{\lambda}m_1^2 + \lambda_1 m_2^2 > 0$  and (b):  $-\hat{\lambda}m_2^2 - \lambda_2 m_1^2 > 0$ , respectively. Two simultaneous ✓ in GM means  $|v_1| \gg |v_2|$  ( $|v_1| \ll |v_2|$ ) vacuum becomes the global minimum when  $-\frac{m_1^4}{2\lambda_1} < -\frac{m_2^4}{2\lambda_2}$  ( $-\frac{m_1^4}{2\lambda_1} > -\frac{m_2^4}{2\lambda_2}$ ).

## 2.2 Stability against radiative corrections

Now we are in a position to investigate the stability of the VEV hierarchy  $|v_2| \ll |v_1|$  against radiative corrections. First of all, we should remind that the small magnitude of  $|m_3^2|$  plays a crucial role for generating the tiny VEV of  $|v_2|$  ( $\ll |v_1|$ ). Its smallness is guaranteed against radiative corrections, since it is the “soft” breaking mass parameter of the  $Z_2$ -symmetry. As noted in Ref.[8], the radiative correction to this parameter is expected to be logarithmic. For analyses of the vacuum stability, we should use Coleman-Weinberg type 1-loop effective potential[18], and analyze the stability of the VEV hierarchy. This 1-loop effective potential contains infinite number of irrelevant operators with zero-momentum Higgs fields in the external lines, and is calculated by a summation of them. However, for the investigation of stability of the VEV hierarchy, it is enough for us to pick up only diagrams which have external lines of mixture of  $\Phi$  and  $\Phi_\nu$ . Furthermore, we should notice that, when one  $\Phi_\nu$  is added in the external lines, a coefficient of the effective operator should have suppression factor,  $|v_2/m_{1,2}|$ . Thus, we investigate diagrams which have only one  $\Phi_\nu$  in the external lines.

At first, we focus on marginal operators in the effective potential. The most dangerous marginal operator for the instability of the VEV hierarchy is  $\lambda_6 |\Phi^2|(\Phi^\dagger \Phi_\nu)$  (+h.c.), which is induced from diagrams in Fig.1 (a) and (b). It is because this operator breaks  $Z_2$ -parity and induces linear term of  $v_2$ , which might possibly destroy the VEV hierarchy. Here we note that Fig.1 (a) and (b) are only 1-loop diagrams which induce  $\lambda_6 |\Phi^2|(\Phi^\dagger \Phi_\nu)$  (+h.c.). Neither lepton nor quark 1-loop diagrams contribute  $\lambda_6$  due to the  $Z_2$ -parity, since one additional external  $\Phi_\nu$  needs one additional right-handed neutrino propagator inside a loop which requires one more  $\Phi_\nu$ . Fig. 1 (c) and (d) induce another  $Z_2$ -parity violating operator,  $\lambda_7 |\Phi_\nu^2|(\Phi^\dagger \Phi_\nu)$  (+h.c.).

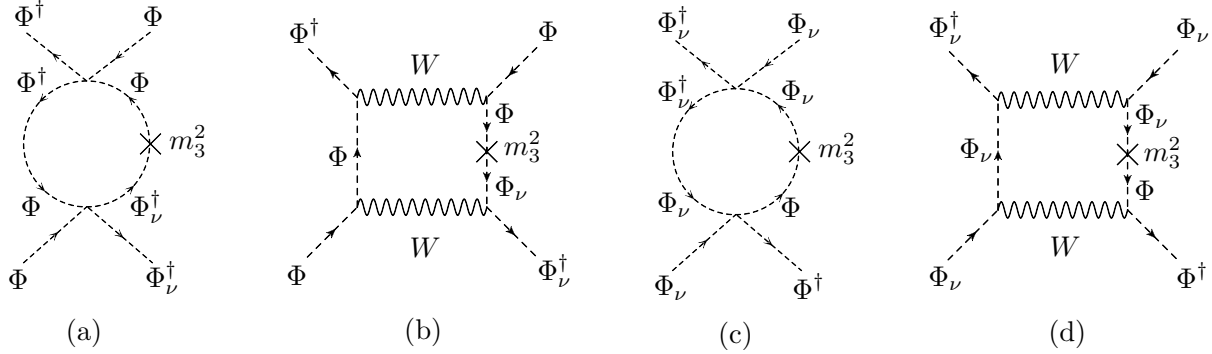


Fig.1:  $Z_2$ -violating 1-loop diagrams.

Figure 1 (a) ((c)) is expected to dominate (b) ((d)) because of  $|\lambda_i|^2 \gg g_2^4$ , so  $\lambda_6$  and  $\lambda_7$  are estimated as

$$\lambda_6 \sim -\frac{3\lambda_1\lambda_5}{4\pi^2} \frac{m_3^2}{(m_2^2 - m_1^2)^2} \left( m_2^2 - m_1^2 + m_2^2 \ln \frac{|m_1^2|}{|m_2^2|} \right), \quad (2.28)$$

$$\lambda_7 \sim \frac{3\lambda_2\lambda_5}{4\pi^2} \frac{m_3^2}{(m_2^2 - m_1^2)^2} \left( m_2^2 - m_1^2 + m_1^2 \ln \frac{|m_1^2|}{|m_2^2|} \right). \quad (2.29)$$

Taking into account all irrelevant operators which have only one  $\phi_\nu$  in the effective operator, correction for  $|\lambda_6|$  might be of order  $\frac{3}{2\pi^2} |\frac{m_3^2}{m_{1,2}^2}| \log |\frac{v_1}{v_2}|$  at most. This correction contributes the stationary condition of  $v_2$  in Eq.(2.4), and modifies it as

$$0 = m_2^2 v_2 - m_3^2 v_1 + \lambda_2 v_2^3 + \hat{\lambda} v_1^2 v_2 + \frac{\lambda_6}{2} v_1^3 + \frac{3\lambda_7}{2} v_1 v_2^2. \quad (2.30)$$

Remind again that tiny VEV of  $|v_2| (\ll |v_1|)$  is originated from tiny term of  $m_3^2 v_1$ . Thus, an induced term from the radiative correction of  $\frac{\lambda_6}{2} v_1^3$  must be smaller than  $m_3^2 v_1$  to preserve the VEV hierarchy. Actually, the ratio of them is estimated as

$$\left| \frac{\lambda_6 v_1^3}{2m_3^2 v_1} \right| \sim \frac{3}{4\pi^2} \log \left| \frac{v_1}{v_2} \right| \quad (2.31)$$

at most. This means that the order of  $|v_2|$  is not changed but its factor might be modified about 0.8 (2) by the radiative corrections in Majorana (Dirac) neutrino scenario. This magnitude comes from a maximal (may be over-) estimation, and anyhow, the orders of VEVs are not changed. (Actually, this modification becomes much smaller about  $\mathcal{O}(1)\%$ , if we use Higgs self-couplings of  $\mathcal{O}(0.1)$ .) Thus, the VEV hierarchy itself is stable against radiative corrections. As for higher-loop effects, they are at least suppressed by an additional loop-factor  $\frac{1}{16\pi^2}$ , and we cannot find any diagrams which have larger contribution than above diagrams. Therefore, the VEV hierarchy itself is stable against radiative corrections, and we can conclude radiative corrections do not destroy the VEV hierarchy in both Dirac and Majorana neutrino scenarios.



### 3 SUSY neutrinophilic Higgs doublet model

In this section, we analyze vacuum structures of Higgs potential in the SUSY neutrinophilic Higgs doublet model at first, and next investigate a stability of this VEV hierarchy against radiative corrections.

#### 3.1 Vacuum structure in tree-level potential

The SUSY neutrinophilic Higgs doublet model has four Higgs doublets[12, 13], and the superpotential is given by

$$\begin{aligned} \mathcal{W} = & y^u \bar{Q}^L H_u U_R + y^d \bar{Q}^L H_d D_R + \bar{L} H_d E_R + y^\nu \bar{L} H_\nu N \\ & + \mu H_u H_d + \mu' H_\nu H_{\nu'} + \rho H_u H_{\nu'} + \rho' H_\nu H_d, \end{aligned} \quad (3.32)$$

where  $H_\nu$  gives Dirac neutrino masses and  $H_{\nu'}$  does not couple with any matters. Note that  $H_u$  and  $H_d$  are usual MSSM Higgs doublets. This superpotential is for Dirac neutrino scenario, and Majorana neutrino scenario can be realized when Majorana mass of right-handed neutrinos  $MN^2$  is included in Eq.(3.32). The  $Z_2$ -parity assignment of the fields is shown in the following table.

| Fields  | $Z_2$ -parity | Lepton number              |
|---|---------------|----------------------------|
| MSSM Higgs doublets $H_u, H_d$                  | +             | 0                          |
| neutrinophilic Higgs doublets $H_\nu, H_{\nu'}$ | −             | 0                          |
| right-handed neutrino $N$                       | −             | 1                          |
| others  | +             | $\pm 1$ :leptons, 0:quarks |

Note that  $Z_2$ -parity is softly broken by  $\rho, \rho'$ , where  $|\rho|, |\rho'| \ll |\mu|, |\mu'|$ . The Higgs potential is given by

$$\begin{aligned} V = & (|\mu|^2 + |\rho|^2) H_u^\dagger H_u + (|\mu|^2 + |\rho'|^2) H_d^\dagger H_d + (|\mu'|^2 + |\rho|^2) H_\nu^\dagger H_\nu + (|\mu'|^2 + |\rho|^2) H_{\nu'}^\dagger H_{\nu'} \\ & + \frac{g_1^2}{2} \left( H_u^\dagger \frac{1}{2} H_u - H_d^\dagger \frac{1}{2} H_d + H_\nu^\dagger \frac{1}{2} H_\nu - H_{\nu'}^\dagger \frac{1}{2} H_{\nu'} \right)^2 \\ & + \sum_a \frac{g_2^2}{2} \left( H_u^\dagger \frac{\tau^a}{2} H_u + H_d^\dagger \frac{\tau^a}{2} H_d + H_\nu^\dagger \frac{\tau^a}{2} H_\nu + H_{\nu'}^\dagger \frac{\tau^a}{2} H_{\nu'} \right)^2 \\ & - m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_{H_\nu}^2 H_\nu^\dagger H_\nu + m_{H_{\nu'}}^2 H_{\nu'}^\dagger H_{\nu'} \\ & + B\mu H_u \cdot H_d + B'\mu' H_\nu \cdot H_{\nu'} + \hat{B}\rho H_u \cdot H_{\nu'} + \hat{B}'\rho' H_\nu \cdot H_d \\ & + \mu^* \rho H_d^\dagger H_{\nu'} + \mu^* \rho' H_u^\dagger H_\nu + \mu'^* \rho' H_\nu^\dagger H_d + \mu'^* \rho H_{\nu'}^\dagger H_u + \text{h.c.}, \end{aligned} \quad (3.33)$$

where  $\tau^a$  and dot mean a generator and cross product of  $SU(2)$ , respectively.  $\hat{B}'\rho'$  ( $\hat{B}\rho$ ) corresponds to  $m_3^2$  in the non-SUSY  $\nu$ THDM, and its smallness plays a crucial role of generating

tiny VEVs of  $H_{\nu,\nu'}$ . The magnitude of  $|\hat{B}'\rho'|$  ( $|\hat{B}\rho|$ ) is  $(\mathcal{O}(10^{-0.5}) \text{ GeV})^2$  for Majorana neutrino scenario, and is  $(\mathcal{O}(10^{-1}) \text{ MeV})^2$  for Dirac neutrino scenario. We assume  $(-m_{H_u}^2) < 0$  for the suitable electroweak symmetry breaking and real VEVs as

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \langle H_\nu \rangle = \begin{pmatrix} 0 \\ v_\nu \end{pmatrix}, \langle H_{\nu'} \rangle = \begin{pmatrix} v_{\nu'} \\ 0 \end{pmatrix}. \quad (3.34)$$

By taking  $\mu, \rho, B$ -parameters to be real and denoting  $M_u^2 \equiv |\mu|^2 + |\rho|^2 - m_{H_u}^2 (< 0)$ ,  $M_d^2 \equiv |\mu|^2 + |\rho'|^2 + m_{H_d}^2 (> 0)$ ,  $M_\nu^2 \equiv |\mu'|^2 + |\rho|^2 - m_{H_u}^2 (> 0)$ , and  $M_{\nu'}^2 \equiv |\mu'|^2 + |\rho|^2 + m_{H_d}^2 (> 0)$ , the stationary conditions are given by

$$\begin{aligned} 0 &= \frac{1}{2} \frac{\partial V}{\partial v_u} = M_u^2 v_u + \frac{1}{4} (g_1^2 + g_2^2) v_u (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B\mu v_d - \hat{B}\rho v_{\nu'} + (\mu\rho' + \mu'\rho) v_\nu, \\ 0 &= \frac{1}{2} \frac{\partial V}{\partial v_d} = M_d^2 v_d - \frac{1}{4} (g_1^2 + g_2^2) v_d (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B\mu v_u - \hat{B}'\rho' v_\nu + (\mu\rho + \mu'\rho') v_{\nu'}, \\ 0 &= \frac{1}{2} \frac{\partial V}{\partial v_\nu} = M_\nu^2 v_\nu + \frac{1}{4} (g_1^2 + g_2^2) v_\nu (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B'\mu' v_{\nu'} - \hat{B}'\rho' v_d + (\mu\rho' + \mu'\rho) v_u, \\ 0 &= \frac{1}{2} \frac{\partial V}{\partial v_{\nu'}} = M_{\nu'}^2 v_{\nu'} - \frac{1}{4} (g_1^2 + g_2^2) v_{\nu'} (v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B'\mu' v_\nu - \hat{B}\rho v_u + (\mu\rho + \mu'\rho') v_d. \end{aligned}$$

Let us investigate the vacuum structure with a parametrization of  $v_u = v \sin \beta \cos \gamma$ ,  $v_d = v \cos \beta \cos \gamma$ ,  $v_\nu = v \sin \beta_\nu \sin \gamma$ ,  $v_{\nu'} = v \cos \beta_\nu \sin \gamma$ . At first, we focus on the vacuum which neutrinophilic Higgs doublet model requires, i.e.,  $|v_u|, |v_d| \gg |v_\nu|, |v_{\nu'}|$ . This condition induces the usual MSSM relations for  $v_u, v_d$  as

$$M_u^2 - \frac{1}{4} (g_1^2 + g_2^2) v^2 \cos 2\beta - B\mu \cot \beta \simeq 0, \quad M_d^2 + \frac{1}{4} (g_1^2 + g_2^2) v^2 \cos 2\beta - B\mu \tan \beta \simeq 0,$$

which means

$$v^2 \simeq \frac{2}{g_1^2 + g_2^2} \left( \frac{M_u^2 - M_d^2}{\cos 2\beta} - (M_u^2 + M_d^2) \right), \quad \sin 2\beta \simeq \frac{2B\mu}{M_u^2 + M_d^2}. \quad (3.35)$$

They induce tiny  $v_\nu, v_{\nu'}$  through tiny  $\rho, \rho'$  as

$$v_\nu = \frac{[M_{\nu'}^2 - \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] [\hat{B}'\rho' v_d - (\mu\rho' + \mu'\rho) v_u] + B'\mu' [\hat{B}\rho v_u - (\mu\rho + \mu'\rho') v_d]}{[M_\nu^2 + \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] [M_{\nu'}^2 - \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] - B'^2 \mu'^2}, \quad (3.36)$$

$$v_{\nu'} = \frac{[M_\nu^2 + \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] [\hat{B}\rho v_u - (\mu\rho + \mu'\rho') v_d] + B'\mu' [\hat{B}'\rho' v_d - (\mu\rho' + \mu'\rho) v_u]}{[M_\nu^2 + \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] [M_{\nu'}^2 - \frac{1}{4} (g_1^2 + g_2^2) (v_u^2 - v_d^2)] - B'^2 \mu'^2}. \quad (3.37)$$

At this vacuum, the potential height is estimated as

$$V \simeq v^2(M_u^2 \sin^2 \beta + M_d^2 \cos^2 \beta - 2B\mu \cos \beta \sin \beta) + \frac{1}{8}(g_1^2 + g_2^2)v^4 \cos^2 2\beta. \quad (3.38)$$

Next, we study the vacuum at  $|v_u|, |v_d| \sim |v_\nu|, |v_{\nu'}|$ . Where, by neglecting both  $\rho$  and  $\rho'$ , the stationary conditions become

$$M_u^2 v_u + \frac{1}{4}(g_1^2 + g_2^2)v_u(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B\mu v_d = 0, \quad (3.39)$$

$$M_d^2 v_d - \frac{1}{4}(g_1^2 + g_2^2)v_d(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B\mu v_u = 0, \quad (3.40)$$

$$M_\nu^2 v_\nu + \frac{1}{4}(g_1^2 + g_2^2)v_\nu(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B'\mu' v_{\nu'} = 0, \quad (3.41)$$

$$M_{\nu'}^2 v_{\nu'} - \frac{1}{4}(g_1^2 + g_2^2)v_{\nu'}(v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2) - B'\mu' v_\nu = 0. \quad (3.42)$$

It is easy to show that only  $v_\nu = v_{\nu'} = 0$  can satisfy the stationary conditions in  $D$ -flat direction of  $v_\nu = v_{\nu'}$ .

Numerical analyzes show that the vacuum at  $v_\nu = v_{\nu'} = 0$  is the global minimum in suitable parameter regions of  $|B'|, |\mu'| = \mathcal{O}(10^2)$  GeV and positive  $M_\nu, M_{\nu'} = \mathcal{O}(10^2)$  GeV. This result is originated from an initial setup that only soft mass squared of  $H_u$  is negative. (See, case (1) of Table 1 in  $\nu$ THDM.) Similarly, we can show that there is no vacuum at  $|v_u|, |v_d| \ll |v_\nu|, |v_{\nu'}|$ . Anyhow, the vacuum exists only at  $|v_u|, |v_d| \gg |v_\nu|, |v_{\nu'}|$ , which is the desirable vacuum in the neutrinophilic Higgs doublet model.

### 3.2 Stability against radiative corrections

Let us investigate the stability of the VEV hierarchy against radiative corrections in the SUSY neutrinophilic Higgs doublet model. As in non-SUSY case, we can estimate 1-loop radiative corrections in a SUSY effective potential.

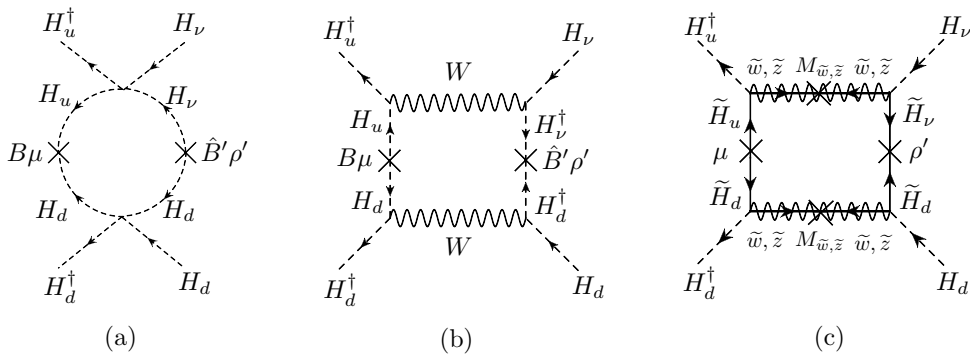


Fig.2:  $Z_2$ -violating 1-loop diagrams in SUSY.

The most dangerous marginal operator in the effective potential is  $\lambda'(H_u^\dagger H_\nu)(H_d^\dagger H_d)$  (+h.c.), which is induced from  $Z_2$ -violating diagrams in Figs.2 (a)~(c). The absolute value of  $\lambda'$  is roughly estimated as  $\frac{g_2^4}{8\pi^2}|\frac{\hat{B}'\rho'}{m^2}|$  at most, where  $m$  is a Higgs mass in a loop. Notice that neither (s)lepton nor (s)quark contribute  $\lambda'$  at 1-loop level due to the  $Z_2$ -parity similarly in non-SUSY  $\nu$ THDM. It is because one additional external  $H_\nu$  needs one additional right-handed neutrino propagator inside a loop, which requires one more external  $H_\nu$ . Anyhow, this term modifies the stationary condition of  $v_\nu$  in Eq.(3.35) as

$$0 = M_\nu^2 v_\nu - \frac{1}{4}(g_1^2 + g_2^2)v_\nu \left[ v_u^2 - v_d^2 + v_\nu^2 - v_{\nu'}^2 + \frac{2\lambda'}{(g_1^2 + g_2^2)} \frac{v_u v_d^2}{v_\nu} \right] - \hat{B}'\rho' v_d + (\mu\rho' + \mu'\rho)v_\nu \quad (3.43)$$

Taking into account all irrelevant operators which have only one  $H_\nu$  in the effective operator, correction for  $|\lambda'|$  might be of order  $\frac{g_2^4}{4\pi^2}|\frac{\hat{B}'\rho'}{m^2}|\log|\frac{v_{u,d}}{v_\nu}|$  at most. Remind that tiny VEV of  $v_\nu$  is originated from the small mass parameters of  $\hat{B}'\rho'$  as in Eq.(3.36). Thus, in order to preserve the VEV hierarchy,  $|\frac{\lambda'}{2}v_u v_d^2|$  must be smaller than  $|\hat{B}'\rho' v_d|$  in Eq.(3.43). And, this ratio is estimated as

$$\left| \frac{\lambda' v_u v_d^2}{2\hat{B}'\rho' v_d} \right| \sim \frac{g_2^4}{8\pi^2} \left| \frac{v_u v_d}{m^2} \right| \log \left| \frac{v_{u,d}}{v_\nu} \right|. \quad (3.44)$$

This value is too small to influence the stationary conditions in both Dirac and Majorana neutrino scenarios. We can also show that higher-loop diagrams induce smaller corrections due to the loop suppression factors. Therefore, we can conclude that the potential is stable against radiative corrections in SUSY neutrinophilic Higgs doublet model.

## 4 Summary

A neutrinophilic Higgs model has tiny VEV, which can naturally explain tiny masses of neutrinos. There is a large energy scale hierarchy between a VEV of the neutrinophilic Higgs doublet and that of usual SM-like Higgs doublet. In this paper, we have analyzed vacuum structures of Higgs potential in both SUSY and non-SUSY neutrinophilic Higgs models, and next investigated a stability of this VEV hierarchy against radiative corrections. We have shown that the VEV hierarchy is stable against radiative corrections in both Dirac neutrino and Majorana neutrino scenarios in both SUSY and non-SUSY neutrinophilic Higgs doublet models.

### Note added

After preparing our submission of this paper, we notice a paper [19], where authors also analyzed the vacuum stability against radiative corrections in the non-SUSY  $\nu$ THDM with

Dirac neutrino scenario. Their results are consistent with ours. They calculated the 1-loop effective potential and the quantum corrections to VEV hierarchy. On the other hand, we estimated the most dangerous contributions to the VEV hierarchy and confirmed the stability also in SUSY and Majorana cases.

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